DAWSON COLLEGE
MATHEMATICS DEPARTMENT
Final Examination

Mathematics 201-NYC-05
Linear Algebra (Regular)
Instructors: Melanie Beck, Yann Lamontagne

Date: Tuesday, May 18, 2010
Time: 2:00 - 5:00

1. Consider the following system of equations:

$$\begin{array}{rcrcrcr} 9x & & 18y & + & 45z & = & 39 \\ 2x & + & 5y & & 11z & = & 28/3 \\ 7x & & 17y & + & 38z & = & 97/3 \end{array} ;$$

   (a) **(5 marks)** Find all solutions using Gauss-Jordan elimination.

   (b) **(2 marks)** Find any two particulars solutions.

2. Consider the matrices

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 1 & 6 \\ 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 3 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 3 \\ 0 & 3 \end{bmatrix}$$

   Compute whenever it is possible:

   (a) **(2 marks)** $A^{\ 1}$

   (b) **(2 marks)** $C^{\ 1}$

   (c) **(2 marks)** $(3A \quad I)B \quad C^T$

   (d) **(2 marks)** $\det(3B)$

   (e) **(2 marks)** $\text{trace}(AC)^T$

3. Let $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 3 & 1 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix}$ and B a $4 \quad 4$ matrix with $\det(B) = 3$.

   (a) **(3 marks)** Find $\det(A)$.

   (b) **(3 marks)** Find $\det(\ 3A^{\ 1}B^2)$.

4. **(5 marks)** Prove that if $A^TA = A$, then A is symmetric and $A = A^2$.

5. **(5 marks)** Show that if a square matrix A satis es $A^2 \quad 4A + I = 0$, then A is invertible and $A^{\ 1} = 4I \quad A$.

6. **(4 marks)** Solve for x: $\begin{matrix} x & 0 & 3 \\ 0 & x+1 & 5 \\ 0 & 0 & 2 \end{matrix} \quad 4 = \begin{matrix} 4 & 4 \\ x & 5 \end{matrix}$

7. Consider the matrices

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 0 & 2 \\ 2 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 0 & 2 \\ 2 & 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3/2 \\ 0 & 2 & 1/2 \\ 0 & 7 & 7 \end{bmatrix}$$

   (a) **(2 marks)** Is it possible to nd an elementary matrix E_1 such that $E_1A = B$? If yes, what is E_1? If no, justify.

   (b) **(2 marks)** Is it possible to nd an elementary matrix E_2 such that $E_2B = C$? If yes, what is E_2? If no, justify.

8. Consider the following system of equations:

$$\begin{array}{rcrcr} 3x & + & 2y & = & 12 \\ x & + & 4y & = & 7 \end{array} ;$$

   (a) **(3 marks)** Solve the system using Cramer's rule.

   (b) **(3 marks)** Solve the system using the inverse of A.

9. Given $u = (3; 0; 1)$, $v = (\quad 2; 1; 2)$ and $w = (4; \quad 2; 1)$, nd

(a) (2 marks) $ku \quad vk$

(b) (2 marks)