Dawson College
Mathematics Department
Final Examination
201-NYB-05 Calculus II - Science
Tuesday, May 24, 2011

Student Name: ______________________________

Student I.D. #: ______________________________

Teacher:

Instructors: M. Chaubey, D. Dubrovsky, Y. Lamontagne, S. Muise, V. Ohanyan, J. Requeima, S. Shahabi, S. Soltuz, O. Veres, O. Zlotchevskaia

TIME: 9:30 – 12:30 (3 hours)

INSTRUCTIONS:

- Print your name and student I.D. number in the space provided above.
- Attempt all questions.
- All questions are to be answered directly on the examination paper.
- Translation and regular dictionaries are permitted.
- [illegible] NON PROGRAMMABLE [illegible]

Question #	*Marks*
1 (20)	
2 (4)	
3 (8)	
4 (4)	
5 (4)	

1. Find each indefinite integral.

a) $\displaystyle\int \frac{3x^2+4x+4}{x^3+x}\,dx$

$$\frac{3x^2+4x+4}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$3x^2+4x+4 = A(x^2+1) + (Bx+C)x$$

if $x=0$, $\boxed{4 = A}$

$\therefore\ 3x^2+4x+4 = 4x^2+4 + Bx^2+Cx$

$\therefore\ 3 = 4+B \Rightarrow \boxed{B=-1}$

$\boxed{4 = C}$

b) $\int \tan^3 x \, dx = \int \tan^2 x \cdot \tan x \, dx$

$= \int (\sec^2 x - 1) \cdot \tan x \, dx$

$= \int \tan x \cdot \sec^2 x \, dx - \int \tan x \, dx$

$= \frac{\tan^2 x}{2} + \ln|\cos x| + C$

c) $\int x^2 e^{4x} \, dx$ $\quad u = x^2 \quad dv = e^{4x} dx$

$\quad du = 2x dx \quad v = \frac{1}{4} e^{4x}$

$= \frac{x^2 e^{4x}}{4} - \frac{1}{2} \int x e^{4x} dx$, $\quad u = x \quad dv = e^{4x} dx$

$\quad du = dx \quad v = \frac{1}{4} e^{4x}$

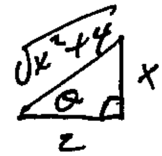
$= \frac{x^2 e^{4x}}{4} - \frac{1}{2}\left(\frac{x e^{4x}}{4} - \frac{1}{4} \int e^{4x} dx\right)$

$= \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{8} + \frac{1}{8}\left(\frac{1}{4} e^{4x}\right) + C$

$= \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{8} + \frac{e^{4x}}{32} + C$

d) $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

$$x = 2\tan\theta$$
$$dx = 2\sec^2\theta\,d\theta$$
$$x^2+4 = 4\sec^2\theta.$$



$$= \int \frac{\cancel{2}\sec^{\cancel{2}}\theta\,d\theta}{4\tan^2\theta\cdot\cancel{\sqrt{4\sec^2\theta}}} = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

e) $\int \frac{\mathrm{d}x}{2+\sqrt{x}}$

$u = 2 + \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$2(u-2)\,du = dx$

$$= \int \frac{2(u-2)\,du}{u}$$

$$= 2\int \left(1 - \frac{2}{u}\right) du = 2\left(u - 2\ln|u|\right) + C$$

$$= 2\left(2+\sqrt{x} - 2\ln|2+\sqrt{x}|\right) + C$$

2. Evaluate the definite integral $\int_0^{\pi} \sin^2(3\theta)\ \mathrm{d}\theta$.

$$= \frac{1}{2}\int_0^{\pi} (1-\cos(6\theta))\,d\theta = \frac{1}{2}\left(\theta - \frac{1}{6}\sin(6\theta)\right)\Big|_0^{\pi}$$

$$= \frac{1}{2}\left[\left(\pi - \cancel{\frac{1}{6}\sin(6\pi)}\right) - \left(\cancel{0} - \cancel{\frac{1}{6}\sin(0)}\right)\right]$$

$$= \pi/2.$$

[redacted]

its value.

a) $\displaystyle\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,\mathrm{d}x$

$u = x^2+2x$ $\quad x=0 \Rightarrow u=0$

$du = (2x+2)dx$ $\quad x=1 \Rightarrow u=3$

$\frac{1}{2}du = (x+1)dx$

$= \frac{1}{2}\int_0^3 \frac{du}{\sqrt{u}}$

$= \frac{1}{2}\lim_{b\to 0^+}\int_b^3 u^{-1/2}\,du = \cancel{\tfrac{1}{2}}\lim_{b\to 0^+}\left(\cancel{2}u^{1/2}\right)\Big|_b^3$

$= \lim_{b\to 0^+}\left(\sqrt{3}-\sqrt{b}\right) = \sqrt{3}$ conv.

4. Find the average value of the function $f(x) = \ln x$ over the interval $[1, e]$.

$f = \frac{1}{\ldots} \int^{e} \ln x \, dx$ $\quad \int \ln x \, dx = x \ln x - \int dx$

$= \frac{1}{e-1} (x \ln x - x) \Big|_1^e$

$= \frac{1}{e-1} \left[(e \ln e - e) - (1 \cdot \cancel{\ln 1} - 1) \right]$

$= \frac{1}{e-1} \left[(\cancel{e - e}) - (-1) \right] = \frac{1}{e-1}$

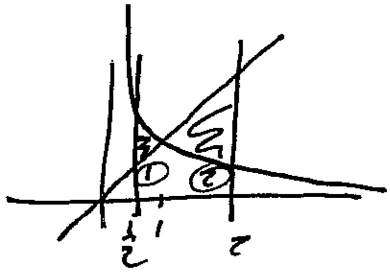
5. Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{\sec x}^{2} \sqrt{1 + t^3} \, \mathrm{d}t$.

$= -\frac{d}{dx} \int_2^{\sec x} \sqrt{1+t^3} \, dt$

6. Use the definition of the definite integral to calculate $\int_2^4 (x^2 - 4x)\, dx$. Verify

$\Delta x = 4-2 \quad 2 \qquad x_i = 2 + 2i$

7. Find the area of the region bounded by the graphs of $y = x$ and $y = \frac{1}{x^2}$, between $x = \frac{1}{2}$ and $x = 2$.



$$① = \int_{1/2}^{1} \left(\frac{1}{x^2} - x\right)dx = \left(-\frac{1}{x} - \frac{x^2}{2}\right)\Big|_{1/2}^{1}$$

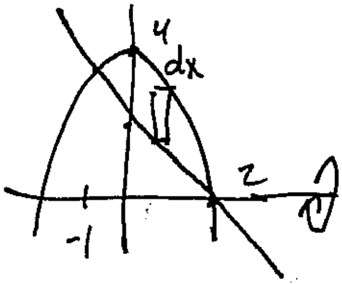
= [redacted]

$$② = \int_{1}^{2} (x - 1/x^2)\,dx = \left(\frac{x^2}{2} + \frac{1}{x}\right)\Big|_{1}^{2}$$

$$= \left[\left(\frac{4}{2} + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right)\right] = 1$$

$$13/8$$

8. Use disks or washers to find the volume of the solid obtained by revolving the region bounded by the graphs of $y = 4 - x^2$ and $y = 2 - x$ about the x-axis.



$R(x) = 4 - x^2$

$r(x) = 2 - x$

$$V = \pi \int_{-1}^{2} \left[(4-x^2)^2 - (2-x)^2 \right] dx$$

$$= \pi \int_{-1}^{2} \left(\cancel{16} - \cancel{8}x^2 + \cancel{x^4} - (\cancel{4} - \cancel{4x} + \cancel{x^2}) \right) dx$$

$$= \pi \int_{-1}^{2} \left(12 \cancel{-} + 4x - 9x^2 + x^4 \right) dx$$

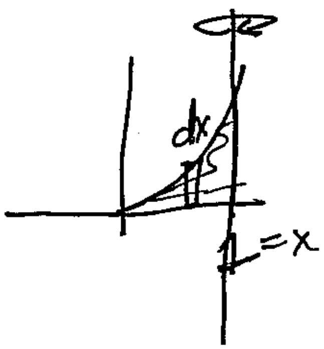
$$= \pi \left(12x \cancel{-} + 2x^2 - 3x^3 + \frac{x^5}{5} \right) \Big|_{-1}^{2}$$

$$= \pi \left[\left(24 \cancel{-} + 8 - 24 + \frac{32}{5} \right) - \left(-12 \cancel{-} + 2 + 3 - \frac{1}{5} \right) \right]$$

$=$

$$\frac{108\pi}{5}$$

9. Use shells to find the volume of the solid obtained by revolving the region bounded by the graphs of $y = 0$ and $y = 3x^4$ between $x = 0$ and $x = 1$ about



$p(x) = 1 - x$

$h(x) = 3x^4.$

$$V = 2\pi \int_0^1 (1-x)(3x^4)\,dx$$

$$= 2\pi \int_0^1 (3x^4 - 3x^5)\,dx$$

$$= 2\pi \left(\frac{3x^5}{5} - \frac{x^6}{2} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{3}{5} - \frac{1}{2} \right) = 2\pi \left(\frac{1}{10} \right) = \pi/5$$

10. Find the arclength of the curve defined by the function $f(x) = \frac{x^3}{12} + \frac{1}{x}$ over the interval $[1, 2]$.

$$+\left(f'(x)\right)^2 = 1 + \left(\frac{x^2}{4} - \frac{1}{x^2}\right)^2 = 1 + \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}$$

$$= \frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4} = \left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2$$

$$\therefore S = \int_1^2 \sqrt{1+f'(x)^2}\,dx = \int_1^2 \left(\frac{x^2}{4} + \frac{1}{x^2}\right)dx$$

$$= \left(\frac{x^3}{12} - \frac{1}{x}\right)\Big|_1^2 = \left(\frac{8}{12} - \frac{1}{2}\right) - \left(\frac{1}{12} - 1\right) = 13/12$$

11. Use the squeeze theorem to find the limit of the sequence $\left\{\frac{2^n}{n!}\right\}$, if it exists.

$$0 < \frac{2^n}{n!} = \frac{2\cdot\cancel{2}\cdot 2 \cdots 2}{1\cdot\cancel{2}\cdot 3 \cdots n} = (2)\cdot\underbrace{\frac{2}{3}}_{<1}\cdot\underbrace{\frac{2}{4}}_{<1}\cdots\underbrace{\left(\frac{2}{n-1}\right)}_{<1}\cdot\frac{2}{n}$$

$$< 2\left(\frac{2}{n}\right) = \frac{4}{n} \to 0 \text{ as } n\to\infty$$

$0 \downarrow 0$

$\downarrow$

0 by squeeze thm

12. Find the sum of each series, if it converges, or show that it diverges.

a) $\sum_{n=1}^{\infty} \frac{3^n - 1}{6^{n-1}} = \sum_{n=1}^{\infty} \frac{3^n}{6^{n-1}} - \sum_{n=1}^{\infty} \frac{1}{6^{n-1}}$

$r = 1/2 < 1$ ✓ $\quad r = 1/6 < 1$ ✓

$a = 3 \quad a = 1$

$= \frac{3}{1 - 1/2} - \frac{1}{1 - 1/6}$

$= \frac{3}{1/2} - \frac{1}{5/6} = 6 - \frac{6}{5} = 24/5$

b) $\sum_{n=2}^{\infty} \ln\left(\frac{n+1}{n+2}\right) = \sum_{n=2}^{\infty} \left(\ln(n+1) - \ln(n+2)\right)$

$S_n = (\ln(3) - \cancel{\ln(4)}) + (\cancel{\ln(4)} - \cancel{\ln(5)}) + \cdots + (\cancel{\ln n} - \cancel{\ln(n+1)}) + (\cancel{\ln(n+1)} - \ln(n+2))$

$S_n = \ln 3 - \ln(n+2)$.

$\therefore \lim_{n \to \infty} S_n = \ln 3 - \infty = -\infty \Rightarrow$ div

13. Determine whether each series converges or diverges. State any tests used to reach your conclusions.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$

$\frac{\sin^2 n}{2^n} \leq \frac{1}{2^n} \leftarrow$ geo with $r = 1/2 < 1$ conv

$\therefore$ conv by DCT.

b) $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)$

$\lim_{n \to \infty} \cos(n\pi) = \text{DNE}$

$\Rightarrow$ div. by Div Test.

c) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$ $\quad f(x) = \frac{1}{x\sqrt[3]{\ln x}}$ $\quad$ pos, cont. decreasing since 1 over increasing

$$\int_2^{\infty} \frac{1}{x(\ln x)^{1/3}}\,dx = \lim_{b\to\infty} \int_{\ln 2}^{b} u^{-1/3}\,du$$

$u = \ln x$
$du = \frac{1}{x}dx$

$$= \lim_{b\to\infty} \left(\frac{3}{2}u^{2/3}\right)\Big|_{\ln 2}^{b}$$

$$= \frac{3}{2}\lim_{b\to\infty}\left(b^{2/3} - (\ln 2)^{2/3}\right) = \infty \Rightarrow \text{div}$$

by integral test.

d) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$ $\quad a_{n+1} = \frac{(n+1)!}{(2(n+1)+1)!} = \frac{(n+1)\cdot n!}{(2n+3)(2n+2)\cdot(2n+1)!}$

$$\therefore \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)\cdot \cancel{n!}}{(2n+3)(2n+2)\cdot\cancel{(2n+1)!}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{n+1}{(2n+3)(2n+2)} = 0 \Rightarrow \text{conv.}$$

ratio test.

14. Determine if the series $\sum^{\infty} (-1)^n$ converges absolutely

16. Find the radius of convergence and the interval of convergence of the power series $\sum^{\infty} \frac{(x-2)^n}{}$ $a \quad = (x-2)^{n+1}$

[redacted]

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(x-2)^n}\right|$$

$$= |x-2|\cdot\lim_{n\to\infty}\sqrt{\frac{n}{n+1}} = |x-2| < 1$$

[redacted]

$$1 < x < 3$$

radius = 1.

if $x=1$, $= \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ conv. by A.S.T.

$\left(b_n = \frac{1}{\sqrt{n}},\ b_{n+1} < b_n,\ \lim_{n\to\infty} b_n = 0\right)$.

if $x=3$, $= \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ div p-series.

$\therefore$ interval $= [1, 3)$.