DAWSON COLLEGE
Mathematics Department
Final Examination
Calculus II
Commerce
201-NYB-05

Thursday, May 19, 2011

Student Name ______________________________

Student I.D. # ______________________________

<u>Instructors</u>: M. Ishii, A. Jimenez, M. Marchant, I. Rajput
<u>Sections</u>: 13, 14, 15, 16

TIME: 9:30 am – 12:30 pm.

Question #	Marks
1/25	

1.) (25 Marks)

Calculate the following integrals

a) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = \int \cos(u)\, 2\sqrt{x}\, du$ (4 Marks)

$dx = 2\sqrt{x}\, du$

b) $\int \sec^2(3x)\tan^3(3x)dx = \int \sec^2(3x)\, u^3\, \frac{1}{}$ (5 Marks)

c) $\int \frac{2x-3}{\sqrt{x-2}}dx = \int \frac{2u+1}{\sqrt{u}}\,du = \int 2\sqrt{u} + \frac{1}{\sqrt{u}}\,du$ (6 Marks)

$= \frac{4}{3}\sqrt{x-2}^{3} + 2\sqrt{x-2} + C$

e) $\int \frac{x-1}{x^2(x-3)}dx$ (5 Marks)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} = \frac{Ax(x-3) + B(x-3) + Cx^2}{x^2(x-3)}$$

$$x-1 = Ax(x-3) + B(x-3) + Cx^2$$

$x = 0$

$-1 = B(-3) \qquad B = \frac{1}{3}$

$x = 3$

$2 = C3^2 \Rightarrow C = \frac{2}{9}$

$x = 1$

$0 = A(-2) + \underbrace{\frac{1}{3}(-2)}_{-\frac{6}{9}} + \frac{2}{9} \Rightarrow A(-2) = \frac{4}{9} \Rightarrow A = -\frac{2}{9}$

$$-\frac{2}{9}\ln|x| - \frac{1}{3}\frac{1}{x} + \frac{2}{9}\ln|x-3| + K$$

2.)
(5 Marks)
Monthly records show that the rate of change for the cost of a product is

$$C^{'}(x)=3\sqrt{2x+25}$$

and the fixed cost for a month is $11 125. What is the total cost of producing 300 items in a month?

$$C(x) = \int C'(x)\,dx = \int 3\sqrt{2x+25}\,dx = \frac{3}{2}\int \sqrt{u}\,du$$

$u = 2x+25$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$$= \frac{3}{2}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \sqrt{u}^{\,3}$$

$$= \sqrt{2x+25}^{\,3} + C$$

When $C(0) = 11\,125$

$$\sqrt{2\cdot 0+25}^{\,3} + C = 5 + C = 11\,125$$

$$C = 11\,120$$

$$C(300) = \sqrt{2\cdot 300+25}^{\,3} + 11\,120 = \sqrt{625}^{\,3} + 11\,120$$

Ans

$26 745

$15625 + 11120$

3.) (5 Marks)

Find the average value of the function $f(x)=xe^{-x^2}$ over the interval [-1, 1]

$$F(x) = \int x e^{-x^2}dx = \int x\, e^{u}\,\frac{1}{-2x}\,dx = -\frac{1}{2}e^{-x^2}$$

$u = -x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{1}{-2x}du$

$$\bar{f} = \frac{F(1) - F(-1)}{1-(-1)} = \frac{-\frac{1}{2}e^{-1} + \frac{1}{2}e^{-1}}{2} = \frac{0}{2} = 0$$

4.) (4 Marks)

Calculate the following integral using only the (Riemman-sums) definition

$\int_0^1 (2x - x^2)\,dx$

Ans: $\frac{2x^2}{2} - \frac{x^3}{3}\Big|_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$ Ans

$$S_n = \sum_{k=0}^{n} \left(\frac{2k}{n} - \frac{k^2}{n^2}\right)\frac{1}{n} = \sum_{k=0}^{n} \frac{2k}{n^2} - \sum_{k=0}^{n} \frac{k^2}{n^3}$$

$$= \frac{2}{n^2}\,\frac{n(n+1)}{2} - \frac{1}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n+1}{n} - \frac{1}{6}\,\frac{n+1}{n}\,\frac{2n+1}{n}$$

$$\lim_{n\to\infty} S_n = 1 - \frac{1}{6}\,1\cdot 2 = 1 - \frac{1}{3} = \frac{2}{3}$$

5.) (5 Marks)

Find the total area of the region enclosed by the functions $f(x)=\sqrt{7x+15}$ and $g(x)=x+3$

$f(x) = g(x)$

$7x+15 = x^2+6x+9$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

$f(0) = \sqrt{15}$

$g(0) = 3$

$f > g$

on $[-2, 3]$

$F(x) = \int \sqrt{7x+15} - x - 3 \, dx = \frac{2}{21}\sqrt{7x+15}^3 - \frac{x^2}{2} - 3x$

$F(3) - F(-2) = \frac{2}{21}\sqrt{36}^3 - \frac{3^2}{2} - 3 \cdot 3 =$

$- \frac{2}{21}\sqrt{7(-2)+15}^3 + \frac{(-2)^2}{2} + 3(-2)$

6.) (4 Marks)
Use Simpson's rule to approximate $\int_0^2 \sqrt{1+x^2}\,dx$ with n=4. The calculations should be done with four decimals precision.

$x_0 = 0 \quad x_1 = \frac{1}{2} \quad x_2 = 1 \quad x_3 = \frac{3}{2} \quad x_4 = 2$

$\Delta x = 2$

$$\int_0^2 \sqrt{1+x^2}\,dx \approx \frac{2}{3}\left(1 + 4\sqrt{1+\tfrac{1}{4}} + 2\sqrt{1+1} + 4\sqrt{1+\tfrac{9}{4}} + \sqrt{1+5}\right)$$

$$= \frac{2}{3}\left(1 + 2\sqrt{5} + 2\sqrt{2} + 2\sqrt{13} + \sqrt{6}\right)$$

$$\approx \frac{2}{3}\left(1 + 4.47213 + 2.82842 + 7.21111 + 2.44948\right)$$

$$= \frac{2}{3} \cdot 17.96114 = 11.97409$$

7.)
(8 Marks)
Calculate the following limits

a) $\lim_{x \to 0} \frac{1-\cos(3x)}{x^2} = \frac{9\,(\cos(0))}{2} = \frac{9}{2}$ (4 Marks)

$$\frac{[1-\cos(3x)]'}{[x^2]'} = \frac{3\sin(3x)}{2x} =$$

$$\frac{[3\sin(3x)]'}{[2x]'} = \frac{9\cos(3x)}{2}$$

b) $\lim\limits_{x\to\infty}\dfrac{e^{4x}+x}{1+e^{5x}} = \lim\limits_{x\to\infty}\left(\frac{4}{5}\,\frac{1}{e^{x}} + \frac{1}{5e^{5x}}\right) = 0$ (4 Marks)

9.)
(4 Marks)
An insurance company is receiving monthly payments of 42 000 dollars. Find the future value of that company 10 years from now with an annual interest rate of 8% compounded continuously.

$$FV = e^{0,08\cdot 10}\int_0^{10} 42\,000\cdot 12\, e^{-0,08t}\,dt$$

Or annuity formula

$$FV = \frac{12\cdot 42000}{0,08}\left(e^{0.08\cdot 10}-1\right)$$

$$= 150\cdot 42000\left(e^{0.8}-1\right)$$

$$\simeq 6300\,000\left(2,2255-1\right)$$

10.)
(5 Marks)

Find the particular solution of the differential equation of $y' = \frac{\ln x}{y}$ subject to the condition that $y(e) = 2$

$$\int y \, dy = \int \ln x \, dx \qquad \text{by parts}$$

$$\frac{y^2}{2} = x \ln x - x + C$$

$$\frac{e^2}{2} = e \ln e - e + C$$

$$\frac{e^2}{2} = C$$

or

$$y^2 = 2x \ln x - 2x + C$$

$$C = e^2$$

etc

Le = C

11.)
(4 Marks)
Find the fourth degree Taylor polynomial of the function

$f(x)=\ln(1-x)$ for values of x close to 0

$f(x) = \ln(1-x) \to f(0) = \ln(1) = 0$

$f'(x) = -\frac{1}{1-x} \to f'(0) = -1$

$f''(x) = \frac{1}{(1-x)^2} \to f''(0) = 1$

$f'''(x) = -\frac{2}{(1-x)^3} \to f'''(0) = -2$

$f^{(4)}(x) = \frac{6}{(1-x)^4} \to f^{(4)}(0) = 6$

$P_4(x) = -x + \frac{x^2}{2} - \frac{1}{3}x^3 + \frac{1}{4}x^4$

12.)
(4 Marks)
Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{2+3^{n+1}}{5^n} = 10$$

$2\sum\left(\frac{1}{5}\right)^n + 3\sum\left(\frac{3}{5}\right)^n = \frac{2}{1-\frac{1}{5}} + \frac{3}{1-\frac{3}{5}} =$

$= \frac{2\cdot 5}{4} + \frac{3\cdot 5}{2} = \frac{5}{2} + \frac{3\cdot 5}{2} = \frac{4\cdot 5}{2} = 10$

13.)
(12 Marks)
Show whether the following series converges or diverges

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n^2+n}$ (4 Marks)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n^2+n} < \sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n^2} = \sum_{n=1}^{\infty} \frac{1}{4n^{3/2}} \quad p = \frac{3}{2} > 1$$

Converges Converges

b) $\sum_{n=1}^{\infty} \frac{5n^2-3n}{7-4n^2}$ Diverges (4 Marks)

$$\lim_{n \to \infty} \frac{5n^2-3n}{7-4n^2} = -\frac{5}{4} \neq 0$$

c) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$ Diverges (4 Marks)

$u = 1 + \ln n$

$\frac{du}{dn} = \frac{1}{n}$

$dn = n\,du$

$F(n) = \int \frac{1}{n} \frac{1}{u} n\,du = \ln|u| = \ln|1+\ln n|$

$\lim_{n \to \infty} \ln|1+\ln n| = \infty$

$F(1) = \ln(1+0) = 0$

14.)
(5Marks)
The demand function for a certain commodity is $p=D(x)=-0.6x^2+160$ and the supply function is [illegible]

15.)
(4 Marks)
An investment is expected to generate income at the rate of R(t) = 1000 dollars per year for the next 20 years. Find the present and future value of this investment if the interest is at 5% per year compounded continuously.

$$FV = e^{0,05\cdot 20}\int_0^{20} 1000\, e^{-0,05\cdot t}\, dt =$$

= or annuity function

$$FV = \frac{mP}{r}\left(e^{rT}-1\right) = \frac{20\cdot 1000}{0,05}\left(e^{0,05\cdot 20}-1\right)$$

$$= 400\,000\,(e-1) = 687312.73$$

<u>INFORMATION SHEET</u>

$$\sum_{k=1}^{n} 1 = n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right]$$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)\ldots+4f(x_{n-1})+f(x_n)\right]$$

$$A = FV = e^{rT}\int^{T} R(t)e^{-rt}\,dt \qquad PV = \int^{T} R(t)e^{-rt}\,dt$$

$$CS = \int_0^{\bar{x}} D(x)\,dx - \bar{p}\,\bar{x} \qquad PS = \bar{p}\,\bar{x} - \int_0^{\bar{x}} S(x)\,dx$$

$$P_n(x) = \sum^{n} f^{(k)}(a)(x-a)^k = f(a) + f'(a)(x-a) + f''(a)(x-a)^2 + \ldots + f^{(n)}(a)(x-a)^n$$