Dawson College
Department of Mathematics
Final Examination
Winter 2011

Calculus I (201-NYA-05)

Date: May 16, 2011, 9:30am   Time: 3 hours

Examiners: J. Requeima,   O. Veres

Student Name: _____

Student I.D.: _____

- Print your name and student ID number in the space provided above.
- All questions are to be answered directly on the examination paper in the provided space.
- Non-programmable calculators are permitted.

This examination consists of 13 questions, 18 pages and 100 points. Please ensure that you have a complete examination before starting. This exam must be returned intact.

Question 1. Evaluate the following limits:

(a) *(4 marks)*

$$\lim_{x}$$

(c) *(4 marks)*

$$\lim_{x \to 3} \frac{x - 3}{4 - \sqrt{13 + x}}$$

Question 2. *(6 marks)* Show that the following function is continuous everywhere:

$$f(x) = \begin{cases} x^2 - 3x + 5 & \text{if} \quad x < 2 \\ 3 & \text{if} \quad x = 2 \\ \dfrac{x^2 + 2x + 7}{x^2 + 1} & \text{if} \quad x > 2 \end{cases}$$

Question 3. *(6 marks)* Find the derivative of $f(x) = 5x - 2x^2$ using the limit definition of the derivative.

Question 4. Find the derivatives of the following functions:

(a) *(4 marks)*

$$f(x) = \frac{x^3}{3} + 3\sqrt{x} - \frac{1}{x} + \sin x$$

(b) *(5 marks)*

$$f(x) = \frac{\sqrt{x^3 - 3x}}{\cos(3x^2 - 1)}$$

(c) *(4 marks)*

$f(x) = x^{1/3} \ln x + \arcsin$

Question 5. *(6 marks)* Find the equation of the tangent line to $f(x) = e^{x^2} \cos x$ at $x = 0$.

Question 6. *(6 marks)* Use logartithmic differentiation to find the derivative of

$$f(x) = (x-2)^{14}(3x^2 + 5x)\sqrt[p]{x^2 - 2}$$

Question 7. *(6 marks)* Find $\frac{dy}{dx}$ given

$$(xy)^3 = 3x^2 y + y$$

Question 8. *(6 marks)* The demand equation for x units of a product is $x = 200 - 0.02p^2$ where p is in the price in dollars.

(a) Find the elasticity of demand function given $E(p) = -\frac{p\,f'(p)}{f(p)}$.

(b) Determine if demand is elastic or inelacstic when $p = \$35$.

(c) At what price is elasticity unitary?

Question 9. *(6 marks)* Suppose the demand equation for a certain product is

$$300x^2 + 13p^2 = 3900$$

where x represents the number of units in thousands demanded each week when the unit price is $p. How much is the quantity demanded increasing when the unit price is $15 per unit and the unit

Question 10. *(5 marks)* Find the domain, intercepts, and any asymptotes of

$$f(x) = \frac{x^2 + x - 2}{x^2 + 2x - 15}$$

Question 11. Compute the following antiderivatives

(a) *(3 marks)*

$$\int \sqrt[3]{x} + 3e^x + \frac{1}{x} \; dx$$

(b) *(3 marks)*

$$\int \frac{x^3 + \sqrt{x} - x}{x} \, dx$$

Question 12. *(8 marks)* A rectangular storage container with an open top is to have a volume of $10m^2$. The length of the base is twice the width of the base. Material for the base costs \$10 per square metre. Material for the sides costs \$6 per square metre. Find the cost of materials for the cheapest such container.

Question 13. Consider the function $f(x) = 3x^4 - 4x^3$

(a) *(1 marks)* Find the domain of f.

(b) *(2 marks)* Find the x and y-intercepts (if any).

(c) *(3 marks)* Find the intervals where f is increasing and the intervals where f is decreasing.

(d) *(1 marks)* Find any relative extrema.

(e) *(3 marks)* Find the intervals where f is concave upward and the intervals where f is concave downward.

(f) *(1 marks)* Find any inflection points.

(g) *(4 marks)* Use the above information to sketch the graph of f. Clearly indicate the coordinates of any points found above.