

Exam

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THIS

(1) Find a power series representation, and its interval of convergence.

(Hint:  $x^2 - 2x = (x-1)^2 - 1$ .)

$$\frac{1}{x^2 - 2x} = \frac{1}{-1 + (x-1)^2} = \frac{-1}{1 - (x-1)^2}$$

The series converges only if  $|x-1| < 1$  and the interval of convergence is  $(0, 2)$ .

(2) Find the exact value of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$ . (Hint:  $\frac{d}{dx} \frac{x}{(1-x)^2} = \frac{1+x}{(1-x)^3}$ )

$$\frac{d}{dx} \frac{x}{(1-x)^2} = \frac{1+x}{(1-x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$\frac{1-x}{(1+x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$x \frac{1-x}{(1+x)^3} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{2^{n/2}}$$

$$\sqrt{2} (\sqrt{2}-1)^4 = \frac{1}{\sqrt{2}} \frac{1-1/\sqrt{2}}{(1+1/\sqrt{2})^3} = \frac{1}{\sqrt{2}} \frac{1-\sqrt{2}}{(1+\sqrt{2})^3}$$

(3) An

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(4) Find

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(5) Find

(6) Comp

$$\left\langle \frac{1-}{1+} \right\rangle$$

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(9) Compute the limit

We have

so that

$$\lim_{(x,y) \rightarrow (0,0)}$$

(10) Show that all tange.

At a point  
the gradient  
and the

$$(-2^{y_0})$$

Since

$$-x_0$$

$$= -x_0$$

$$= 0,$$

any

(11)

(12)

8 7

(13) Find

at

at

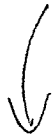
(14) Find



(15) Find the (absolute) mi

Over the disc

$$z^{-2} =$$



the min value  
attained at  
point of  $t$   
circle with

(16) Compute the integral  $\int$

By Fubini

(17) Compute the integ

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \arcsin x \Big|_0^1 = \frac{\pi}{2}$$

(18) Compute the volun

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2} dy dx = \int_0^1 \left[ \frac{1}{2} (1-x^2-y^2)^{3/2} \right]_0^{\sqrt{1-x^2}} dx = \int_0^1 \frac{1}{2} (1-x^2)^{3/2} dx$$

$$= \frac{1}{2} \int_0^1 (1-x^2)^{3/2} dx = \frac{1}{2} \left[ \frac{x}{8} (1-x^2)^{3/2} + \frac{3}{8} \arcsin x \right]_0^1 = \frac{1}{2} \left( \frac{1}{8} (1-1)^{3/2} + \frac{3}{8} \arcsin 1 \right) = \frac{1}{2} \left( 0 + \frac{3}{8} \frac{\pi}{2} \right) = \frac{3\pi}{32}$$

(19) Use a double integral to find the area of the circle  $x^2 + y^2 = 4$ .

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the region  
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(20) Compute the volume of a sphere of radius 1. (Hint: use spherical coordinates, which  $dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$ .)

The integral  
 $\int_0^{2\pi} \int_0^\pi \int_0^1$

$$= 2\pi$$

$$= 2\pi$$

$$=$$

$$=$$