

DAWSON COLLEGE

DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

CALCULUS-III

May 24, 2016

Time: 2:00 pm-5:00 pm

Instructor: A. Panait, T. Kengatharam

Name:

ID:

Instructions:

- Translation and regular dictionaries are permitted.
- Scientific non-programmable calculators are permitted.
- Print your name and ID in the provided space.
- This examination booklet must be returned intact.

This examination consists of 20 questions. Please ensure that you have a complete examination before starting.

(3) [5 marks] Approximate the sum of the convergence series ¹

(4) [5 marks] Evaluate the integral $\int_0^1 xe^{-x^3} dx$ as an infinite series. (Hint: You may use $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$)

- (5) [5 marks] Consider the curve with parametric equations $x = e^t; y = te^t$.
Find $\frac{dy}{dx}$. For which values of t is the curve concave upward?

- (6) [5 marks] Sketch the curve with polar equation $r = 1 - \cos \theta$ for $0 \leq \theta < 2\pi$.

(7) [5 marks] Find the equation of the tangent line to the curve with parametric equations $x = 1 + \sqrt{t}$; $y = e^{t^2}$ at the point $(2; e)$.

(8) [5 marks] Find the arc length of the curve $\underline{r}(t) = (\cos t; \sin t; \ln(\cos t))$ for $0 \leq t \leq \pi/4$.

(9) [5 marks] Show that the curvature of a circle with radius a is $\frac{1}{a}$.

(10) [5 marks] Find the equation of the osculating plane to the curve $\underline{r}(t) = (t; t; t)$ at $(1; 1; 1)$.

(11) [5 marks] Study the continuity of

$$f(x; y) = \begin{cases} \frac{xy - y}{x^2 + y^2} & \text{if } (x; y) \neq (0; 0) \\ - & \text{if } (x; y) = (0; 0) \end{cases} .$$

- (13) [5 marks] Find all critical points of $f(x; y) = 3y^2 - 2y - 3x + 6xy$ and classify them.

- (14) [5 marks] If a particle with mass m moves with position vector $\underline{r}(t)$, then its angular momentum is defined by $\underline{L}(t) = m\underline{r}(t) \times \underline{v}(t)$ and its torque as $\underline{\tau}(t) = m\underline{r}(t) \times \underline{a}(t)$, where $\underline{v}(t)$ and $\underline{a}(t)$ are the particle's velocity and acceleration respectively. Show

- (17) [5 marks] Find the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

- (18) [5 marks] Compute the volume of the tetrahedron bounded by the plane $x + 2y + 3z = 6$ and the three coordinate planes.

- (19) [5 marks] Using cylindrical coordinates evaluate $\int \int \int_E x \, dv$ where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z = 4x + 4y$.

(20) [5 marks] Prove that $\int \int \int_E z e^{(x^2+y^2+z^2)^6} dV \leq 0$, where E is the lower hemisphere $\{(x; y; z) \mid x^2 + y^2 + z^2 \leq 1; z \leq 0\}$.

(Hint: you may use the spherical coordinates $x = \sin \theta \cos \phi$; $y = \sin \theta \sin \phi$; $z = -\cos \theta$ for which $dV = \sin \theta \, d\theta \, d\phi \, dr$.)