DAWSON COLLEGE
Mathematics Department
Final Examination
Linear Algebra
201ïNYCï05 (Commerce)
Fall 2019

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.
   b) (1 mark) Find a particular solution in which $x_1 \quad 3$ and $x_3 \quad 5$.

$$x_1 \quad x_2 \quad 2x_3 \quad x_4 \quad 7$$
$$2x_1 \quad x_2 \quad 2x_3 \quad 8$$
$$3x_1 \quad 2x_2 \quad 4x_3 \quad x_4 \quad 15$$

2. (6 marks) Given the system of linear equations $\begin{matrix} 2x & y & z & 1 \\ x & 2y & 2z & 1 \\ 3x & 2y & 5z & 5 \end{matrix}$.

a) Use the adjoint matrix to find the inverse of the coefficient matrix.
b) Use the inverse of the coefficient matrix to solve the system.

3. (3 marks) Use Cramerôs rule to solve the system from question #2 **<u>for ñzò only</u>**.

4. (3+1+4 marks) Let $A \quad \begin{matrix} 7 & 3 \\ 2 & 1 \end{matrix}$, $B \quad \begin{matrix} 2 & 1 \\ 5 & 2 \\ 1 & 3 \end{matrix}$ and $C \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$.

a. Calculate $tr(A^{\ 2} \quad 3B^T B \quad 4C^{2019})$

b. Does the system $B^T X \quad 0$ have a nontrivial solution? Justify your answer without solving the system.

c. Solve for X : $AX^T \quad 2I^{\ 1} \ ^{1} \quad A^2$

5. (3 marks) Determine the values of a such that the system has
   1) a unique solution, 2) infinitely many solutions, 3) no solution :

$$x \quad 2y \quad 4az \quad 3$$
$$y \quad 7az \quad 2$$
$$x \quad 3y \quad a^2 \quad 2a \ z \quad a \quad 6$$

6. (3 marks) Let

a) $\begin{vmatrix} d & mg & 3a & g & & g & a \\ e & mh & 3b & h & & h & b \\ f & mi & 3c & i & & i & c \end{vmatrix}$, where m is a real number.

b) det $\quad 2B^3 \quad ^1 \quad B^T A \ ^1 \ ^4$

c) det $\quad A\ ^1 B \quad ^1 \quad B\ ^1 \ adj(A\ ^1)$

10. (3+3+3+3 marks) Let $\vec{u} \quad 3,\ 2,\ \ 1\ \ ,\ \vec{v} \quad 1,\ 2,\quad$ and $\vec{w} \quad \vec{i} \quad \vec{j} \quad 2\vec{k}$

a) Find the area of the triangle determined by $\vec{u} \quad \vec{w}$ and $\vec{u} \quad \vec{v}$.

b) Find a vector of length 5 perpendicular to both vectors $\vec{u} \quad \vec{w}$ and $\vec{u} \quad \vec{v}$.

c) Find $Proj_{\overline{u}\ \overline{v}}\ \vec{u} \quad \vec{w}$

d) Find the value(s) of k such that the vector $\vec{v} \quad k\vec{w}$ is perpendicular to $\vec{v} \quad k\vec{u}$.

11. (3 marks) Let $\vec{a} \quad \vec{c} \quad \vec{a} \quad \vec{c} \quad \vec{a} \quad \vec{b} \quad\quad 4$. Find the volume of the parallelepiped with edges $\vec{a}, \vec{b}, \vec{c}$.

12. (1+3 marks) a) Determine whether the planes $x \quad 2y \quad z \quad 2$ and $2x \quad 3y \quad 5z \quad 1$

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Answers

1. a) $x_1 \quad 1 \quad t,\ x_2 \quad 6 \quad 2s \quad 2t,\ x_3 \quad s,\ x_4 \quad t.$ b) $x_1 \quad 3,\ x_2 \quad 8,\ x_3 \quad 5,\ x_4 \quad 2.$

2. a) $A^{1} \quad \begin{matrix} \frac{6}{7} & 1 & \frac{4}{7} \\ \frac{1}{7} & 1 & \frac{3}{7} \\ \frac{4}{7} & 1 & \frac{5}{7} \end{matrix}$; b) $x \quad 1,\ y \quad 1,\ z \quad 2.$

3. $z \quad 2$

4. a) 94; b) Yes, it does. c) $X \quad \begin{matrix} 29 & 70 \\ 105 & 251 \end{matrix}$.

5. 1) $a \quad 0,\ a \quad 1;$ 2) $a \quad 1;$ 3) $a \quad 0$

6. $A^{1} \quad \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & \frac{1}{3} \end{matrix} \quad \begin{matrix} 1 & 0 \\ 2 & 1 \end{matrix}$. *Other possible answers.*

7. $BC \quad CB$

8. 80.

9. a) 8; b) $\frac{1}{32}$; c) $\frac{1}{16}$

10. a) $\sqrt{19}$; b) $\frac{15}{\sqrt{19}}, \frac{5}{\sqrt{19}}, \frac{15}{\sqrt{19}}$; c) $\frac{5}{2}, 0, \frac{5}{2}$; d) $k \quad 2,\ k \quad 3.$

11. 2

12. a) not parallel; b) .

13. a) $\frac{\sqrt{910}}{\sqrt{61}}$; b) $15x \quad 26y \quad 3z \quad 71 \quad 0$; c) $0,1,7$; d) $1, \quad 1,10$.

14. $2\sqrt{5}$

15. True;

16. $P \quad 27,\ x_1 \quad 0,\ x_2 \quad 5,\ x_3 \quad 7.$

17. $C \quad 14,\ x_1 \quad 7,\ x_2 \quad 0.$